Heavy ion collisions in the used nucleon model

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Abstract

It is shown that recently proposed by R.J. Glauber the used nucleon model combined with the assumption that the nucleon consists of two constituents (a quark and a diquark) describes well the PHOBOS data on particle production at midrapidity.

1. In this note we investigate the consequences of the used nucleon model [1] in the context of the PHOBOS data on particle production at midrapidity [2–4]. This model was formulated as a simple generalization of the wounded nucleon model [5] which fails to describe correctly the observed particle multiplicities.

Contrary to the wounded nucleon model, we assume that the number of particles produced from one wounded nucleon does depend on the number of inelastic collisions this nucleon underwent. To formulate this model in more detail let us, for a while, consider nucleon-nucleus (mass number A) collision. For the incident nucleon contribution we have: the first inelastic collision produces n particles, where $2n \equiv (dn/d\eta)|_{|\eta|<1}$ is the average multiplicity at midrapidity in a single proton-proton collision. Following [1] we assume that the second collision produces a fraction μ of n, the third one μ^2, and so on. The number of particles produced (from the incident nucleon) after k collisions is:

$$n_k \equiv n\left(1+\mu+...+\mu^{k-1}\right) = n\frac{1-\mu^k}{1-\mu}. \tag{1}$$

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From (1) it is seen that the used nucleon model gives a natural way of interpolating between the two limits: for $\mu = 0$ we arrive at the wounded nucleon model [5] i.e. there is no difference whether a nucleon is hit once or several times, $n_k = n$; for $\mu = 1$ the incident nucleon contribution is proportional to the number of collisions, $n_k = nk$.

The probability P_k that the incident nucleon underwent exactly k inelastic collisions is given by a standard formula:

$$P_k(b) = \binom{A}{k} \left[\sigma_{NN} T_{NN}(b)\right]^k \left[1 - \sigma_{NN} T_{NN}(b)\right]^{A-k}, \tag{2}$$

where T_{NN} is given by:

$$T_{NN}(b) = \frac{1}{\sigma_{NN}} \int \sigma_{NN}(s)\, T(b-s) d^2 s. \tag{3}$$

Here σ_{NN} and $\sigma_{NN}(s)$ are the total and differential inelastic proton-proton cross-sections, respectively. $T(s)$ is the thickness function (normalized to unity).

The average number of particles produced from the incident nucleon is given by:

$$\sum\nolimits_k P_k n_k = \frac{n}{1-\mu} \left\{1 - \left[1 - (1-\mu)\, \sigma_{NN} T_{NN}(b)\right]^A\right\}. \tag{4}$$

Consequently, the average multiplicity at midrapidity $N \equiv (dN/d\eta)|_{|\eta|<1}$ for the symmetric nucleus-nucleus collision at a given impact parameter b one obtain

$$N(b) = \frac{2A}{\sigma_{AA}(b)} \frac{n}{1-\mu} \int T(b-s) \left\{1 - \left[1 - (1-\mu)\, \sigma_{NN} T_{NN}(s)\right]^A\right\} d^2 s, \tag{5}$$

where $\sigma_{AA}(b)$ is the inelastic differential nucleus-nucleus cross-section.[1]

2. The PHOBOS data are presented versus the number of wounded nucleons in both colliding nuclei W, given by [5]

$$W(b) = \frac{2A}{\sigma_{AA}(b)} \int T(b-s) \left\{1 - \left[1 - \sigma_{NN} T_{NN}(s)\right]^A\right\} d^2 s, \tag{6}$$

Here and in (5) for the nuclear density we use the standard Woods-Saxon formula with the nuclear radius $R_{Au} = 6.37$ fm and $d = 0.54$ fm. We assume the differential proton-proton cross-section $\sigma_{NN}(s)$ to be in a Gaussian form:[2]

$$\sigma_{NN}(s) = \gamma e^{-s^2/\varkappa^2}, \tag{7}$$

[1]In case of Au-Au collisions, using the optical approximation, we have verified that $\sigma_{AA}(b) = 1$, for $b < 14$ fm.

[2]We performed the full calculation also in case of point-like interaction approximation, $\sigma_{NN}(s) = \sigma_{NN}\delta^2(s)$. We have observed that centrality dependence is slightly worse, indicating that the more complete treatment is indeed needed.

where $\varkappa^2 = \sigma_{NN}/(\pi\gamma)$ and $\gamma = 0.92$ [6, 7].

We have observed that the used nucleon model gives a good description of the RHIC Au-Au data with $\mu \sim 0.41$.

The comparison of the used nucleon model ($\mu = 0.41$) with the PHOBOS data [2–4] on the average multiplicity at midrapidity per one wounded nucleon as a function of W is shown in Fig. 1. Also shown are inelastic pp data (points at $W = 2$), as quoted in [3, 4].

The inelastic proton-proton cross-sections needed for this calculation were taken as $\sigma_{NN} = 32$ mb, 36 mb, 41 mb and 42 mb at the RHIC energies $\sqrt{s} = 19.6$, 62.4, 130 and 200 GeV, respectively.

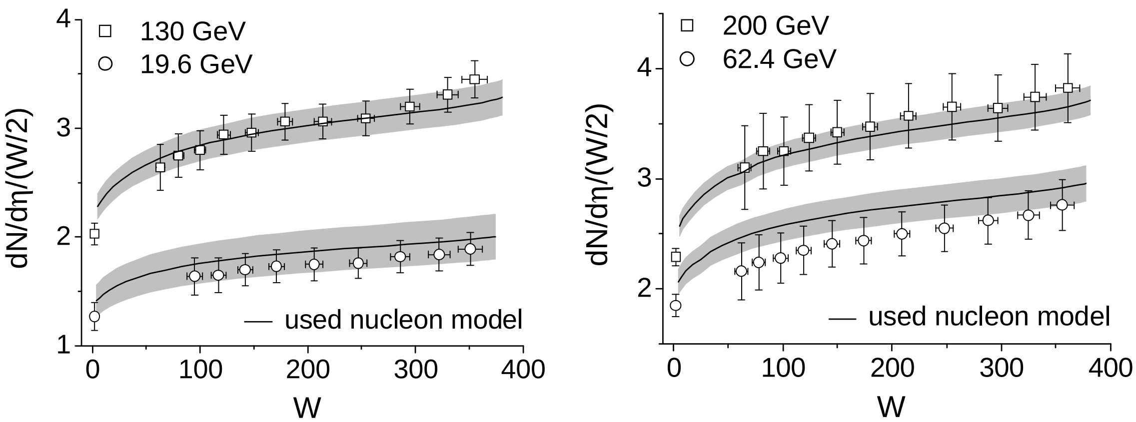

Figure 1: The predictions of the used nucleon model (for $b \leq 14$, corresponding to $W \gtrsim 5$) with $\mu = 0.41$ compared with the PHOBOS data [2–4]. The shaded areas reflect the inaccuracy in the inelastic pp data.

3. To interpret the parameter μ let us consider two simple production scenarios.

(i) Wounded quark model [8]. It is assumed that the nucleon consists of three constituent quarks and particle production from these quarks is independent on the number of interactions they underwent. Furthermore, we assume that the number of produced particles after k collisions is proportional to the number of wounded quarks, that is:

$$\frac{n_k}{n} = \frac{w_k^{(q)}}{w^{(q)}}, \tag{8}$$

where $w^{(q)}$ is the average number of wounded quarks per nucleon in a single inelastic proton-proton collision, $w_k^{(q)}$ is the average number of wounded quarks in the nucleon which underwent k inelastic collisions. The latter may be calculated by a straightforward counting of probabilities:

$$w_k^{(q)} = 3\left[p_q + p_q(1-p_q) + ... + p_q(1-p_q)^{k-1}\right]. \tag{9}$$

Here p_q is the probability for a quark to interact in a single proton-proton collision (note that $p_q = w^{(q)}/3$). Taking (8) and (9) into account we obtain:

$$n_k = n\left[1 + (1-p_q) + ... + (1-p_q)^{k-1}\right] = n\frac{1-(1-p_q)^k}{p_q}. \tag{10}$$

Comparing (10) with (1) we find that $\mu = 1 - p_q$.

It turns out that at the RHIC energies approximately $w^{(q)} \approx 1.2$ [9], giving $\mu \approx 0.6$. We checked that this value leads to significantly larger multiplicities than actually observed [10].[3]

(ii) Wounded quark-diquark model [7]. We assume that the nucleon is composed of two constituents (a quark and a diquark). As above we assume that particle production from these constituents is independent on the number of interactions they underwent, and both constituents produce the same number of particles. Performing analogous calculations as above we obtain:

$$n_k = n\frac{1-(1-p_q)^k}{p_q+p_d} + n\frac{1-(1-p_d)^k}{p_q+p_d}, \tag{11}$$

were p_q and p_d are the probabilities for a quark and a diquark to interact in a single proton-proton collision, respectively.

As it is not clear what the diquark is, however, we may consider two different possibilities:

(a) $p_d = p_q$, it corresponds to the situation where both constituents are *similar*.[4] In this case (11) reduces to (1) with $\mu = 1 - p_q$. Assuming the average number of wounded constituents in a single proton-proton collision $w^{(q+d)} = p_q + p_d$ to be $1.18 \div 1.19$ [7] we obtain the proper value $\mu \approx 0.41$.

(b) $p_d = 2p_q$, where diquark is rather large, comparable to the size of the proton (this relation was obtained in [7]). Now it is not possible to use formula (5), however, performing analogous calculations leading to (5) we obtain:

$$\begin{aligned} N(b) = \frac{2A}{\sigma_{AA}(b)}\frac{n}{w^{(q+d)}}\int T(b-s)\Big\{&1-\left[1-p_q\sigma_{NN}T_{NN}(s)\right]^A + \\ &+ 1-\left[1-p_d\sigma_{NN}T_{NN}(s)\right]^A\Big\}\, d^2s. \end{aligned} \tag{12}$$

We have checked that this formula with $p_d = 2p_q$ and $w^{(q+d)} = 1.185$ [7] gives the results which practically do not differ (less than 5 %) than those presented in Fig. 1. Thus, we may conclude that the model is almost independent on the details of the diquark[5], and the only thing that really matters is the number of constituents.

[3]In order to obtain the value of $\mu \simeq 0.41$ we would have to assume that $w^{(q)} \approx 1.75$. This number, however is difficult to justify.

[4]It may mean that both constituents are of the same size.

[5]We also checked for other choices between $p_d = p_q$ and $p_d = 2p_q$.

4. In conclusion, we have shown that the used nucleon model combined with the assumption that the nucleon consists of two constituents (a quark and a diquark) naturally describes the PHOBOS data on particle production at midrapidity.

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